

FORM OF OPHTHALMIC LENSES

Spectacle lenses are numbered in terms of their back vertex powers, which, when the thickness of a lens is ignored, is the same as the sum of its two surface powers. Thus, if the front surface power of a lens is +9.00 and the back surface power is -5.00, then the total power of the lens is +4.00D. In theory, we could make a +4.00D lens in any of the forms shown in Figure 1. The first two forms in the figure each have a pair of convex surfaces and are known as *flat form lenses*. The third form which has one plane surface is also a flat-form lens and is known as a *plano-convex* lens form. The remaining forms in the figure each have one convex surface and one concave surface and are known as *curved* or *meniscus* lenses.

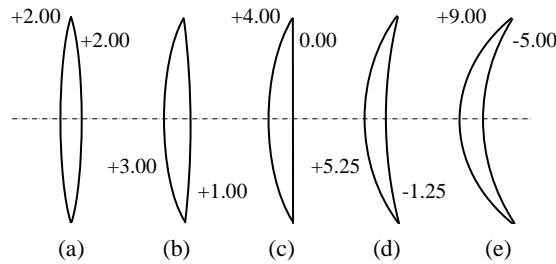


Figure 1 The form of plus spectacle lenses

Figure 2 illustrates various forms in which a -4.00D lens might be made. Once more, the first three forms are flat-form designs and the remainder are of curved form. The lower numerical surface power of a curved lens is known as the *base curve* and the designs illustrated in figures 1(e) and 2 (e) are described as *5.00 base meniscus* lens forms.

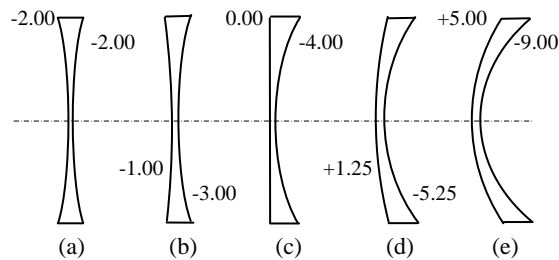


Figure 2 The form of minus spectacle lenses

When a spectacle lens is mounted in front of the eye in such a position that the optical axis of the lens coincides with the visual axis, the form of the lens does not matter. The eye is then viewing through the optical centre of the lens and the image formed by the lens is not afflicted with any defects or aberrations which might affect its sharpness or its shape. Figure 3 summarises the correct centration of a spectacle lens for either distance or near vision. For distance vision, in the absence of any prescribed prism, the optical centres lie on a line which passes through the centre of the pupil and in the vertical meridian the height has been adjusted to take into account the pantoscopic angle of the frame.

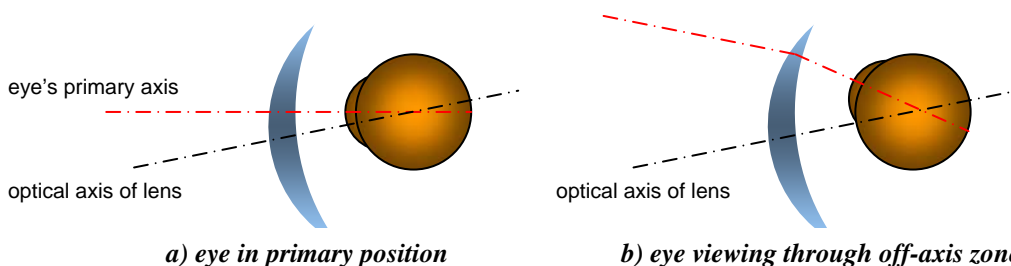


Figure 3 Off-axis vision through a spectacle lens

Although the lens in Figure 3 has been correctly centred for one direction of gaze, in practice, the eye turns behind the lens to view through off-axis visual points. It is under these circumstances that the form in which a lens is made assumes importance. Ideally, the off-axis performance of the lens should be the same as the optical performance of the lens at the optical centre. In general, the off-axis performance of the lens is not the same as its performance along the optical axis, the off-axis images are afflicted with various aberrations which spoil the quality of the images formed by the lens.

The aberrations which are of significance to the spectacle wearer are:-

- transverse chromatic aberration
- oblique astigmatism
- curvature of field
- distortion.

The effects of these aberrations upon image quality is best understood by viewing the target shown in Figure 3 through a strong plus lens such as a trial lens in the range, +12.00 to +16.00D, made in plano-convex form. The appearance of the figures viewed through the lens is described below and demonstrates well the effects of the individual aberrations which might be reported by spectacle wearers.

Transverse Chromatic Aberration

Although transverse chromatic aberration (TCA) is due to the material of the lens it is convenient to deal with it here. The effect of TCA is to cause coloured fringes to be seen surrounding the image of a high contrast target.

For example, when Figure 2.2 is viewed through a strong plus lens held close to the eye, the figure being focussed by bringing the page slowly towards the lens with the centre of the target central in the lens aperture, coloured fringes will be seen around the circular lines.

The order of colours and their positions should be noted. When light from a small white object is refracted by a prism, the light is dispersed into its monochromatic constituents, the blue wavelengths being deviated more than the red. To an eye viewing through the prism the image of the object appears fringed with blue on the apex side of the prism. If the vertical line that forms the 90 meridian of the target in figure 2.2, passes through the optical centre of the lens no fringes will be seen on either side of the vertical line. However, a yellow-red fringe can be seen on the outer edge of the circular lines and blue-cyan on the inner edge of the circular lines. It will be realised that it is the white paper between the lines which acts as the source. The dispersion increases as the rings get further from the optical centre, the fringing is most easily seen on the outermost ring.

Under conditions of low contrast, colour fringing may not be noticed. Instead, the effect of TCA is to cause a reduction in visual acuity. This effect is usually referred to as off-axis blur and often gives rise to the complaint "*the lenses are fine when I look through the centres but are blurred when I look through the edge*".

TCA is due to the dispersive power of the lens, that is, the fact that the refractive index of the lens material varies with the wavelength of the incident light. It is usual to express the dispersive power by its reciprocal, the *constringence* of the material, denoted by the *V-value* or *Abbe Number* for the material.

To a good approximation the magnitude of the TCA at any given point on a lens can be found by calculating the prismatic effect, P , at that point and dividing by the V -value of the material, that is, $TCA = P/V$. It is generally considered that the average threshold value for TCA is 0.1Δ . TCA less than 0.1Δ is unlikely to give rise to complaints. The V -value for materials whose refractive index is in the region of 1.5 (eg, crown glass and CR 39), is about 60 and the prismatic effect at the visual point would need to be about 6Δ before the typical threshold is reached. Using paraxial theory, this amount of prism would be encountered, for example, at a point 15mm from the optical centre of a +4.00D lens.

Materials whose V -values are in the region of 40 would give rise to 0.1Δ of TCA at a point where the prismatic effect is 4Δ , ie 10mm away from the optical centre of a +4.00D lens. It is for this reason that it is wise to select a material with the highest possible V -value.

Oblique Astigmatism

When a narrow pencil of rays is refracted obliquely by a spherical surface the refracted pencil becomes astigmatic. Instead of the rays re-uniting in a single image point, they form two separated line foci at right angles to one another with a disk of least confusion, where the refracted pencil has its least cross-sectional area, somewhere between the two foci. The plane which contains the optical

axis of the surface is referred to as the tangential plane and the plane at right angles to the tangential plane is referred to as the sagittal plane.

Figure 2.3 illustrates the tangential and sagittal meridians for three narrow pencils of rays incident at different points on the surface. The focal lines formed by the refracted pencil are at right angles to the meridian of corresponding power.

As a rule, the tangential power of a lens worn close to the eye, for a given oblique pencil, is in excess of the sagittal power and both are generally greater than the paraxial power. The effect of oblique astigmatism is to produce a blurring of the image as though an unwanted sphero-cylinder had been interposed between the lens and the eye.

Figure 2.4 shows the passage of a narrow pencil of oblique rays through a plus lens mounted before the eye. The refracted pencil is afflicted with aberrational astigmatism and the tangential focus, T' , lies closer to the lens than the sagittal focus, S' . Ideally, the lens should produce a point image of a distant point object on the eye's far point sphere. The terminology used for off-axis imagery is summarised in the caption to Figure 2.4. The vertex sphere is an imaginary reference surface concentric with the eye's centre of rotation from which the positions of the tangential and sagittal foci are measured. The far point sphere is the imaginary surface, also concentric with the eye's centre of rotation, upon which we can assume the far point to remain as the eye rotates to view through off-axis zones of the lens. The distance between the vertex sphere and the far point sphere measured through the eye's centre of rotation, Z , is constant and equal to the back vertex focal length of the lens, A_2F' .

A good idea of the effect of oblique astigmatism upon the eye can be obtained by studying the target depicted in Figure 2.2 through a strong plus lens.

Lines in a radial direction correspond to tangential meridians and so will be seen clearly when sagittal fans of rays are in focus on the retina. Similarly, the concentric circles form the sagittal meridians of the target and will be seen clearly when tangential fans of rays are in focus.

Again, the target should be held well beyond the focusing distance and brought slowly towards the lens which itself should be held close to the eye.

It will be seen that the first area of the target to come into focus is the central zone. This indicates that the power of the lens is weakest in the paraxial region.

As the target is brought closer still to the eye, so each successive zone will come into and go out of focus, showing that the power of the lens increases gradually and continuously towards the periphery. It will also be observed that, for any given zone, the radial line in the neighbourhood of that zone comes into focus before the concentric circle, confirming that the tangential power is in excess of the sagittal power for the zone.

If the target is being observed through a plano-convex lens it will be noticed that the difference in the clarity of the lines at each position is much greater (i.e., the astigmatism is much greater) when the curved surface of the lens is placed next to the eye, than when the plane surface is next to the eye.

The reduction of oblique astigmatism is very important in the design of spectacle lenses and it will be seen that this may be achieved by a suitable choice of lens bending or by the use of an aspherical surface.

Curvature of Field

Figure 2.2 can also be used to detect the presence of curvature of field. It was noted in the above demonstration of the effects of oblique astigmatism, that when the central zone of the target was in focus, the outer zones were not.

If the central zone is now brought into focus and attention turned to the horizontal radial line (along the 180 meridian of the target), it will be noted that the circular rings become progressively out of focus as the eye rotates away from the centre of the target.

If the page is now flexed, so that it presents a vertical concave cylindrical face to the eye (by bending the horizontal edges of the page towards the eye), it will be noted that all the rings along the 180 line are in focus at the same time. The curvature of the object field is being made to match, mechanically, the desired curvature for the image plane. As a rule, when the aberration, oblique astigmatism is corrected, the effects of curvature of field are also reduced.

Field diagrams

A most useful guide to the effects of oblique astigmatism and curvature of field in a given spectacle lens is obtained by studying a field diagram for the lens form. A field diagram (Figure 2.5) is a plot of the tangential and sagittal oblique vertex sphere powers against the ocular rotation of the eye viewing through the lens. In the case of a perfect lens, such as the +4.00 design whose ideal field diagram is

illustrated in figure 2.5a, the tangential and sagittal oblique vertex sphere powers remain +4.00 for all zones of the lens. Unfortunately, this performance is impossible to obtain in a single lens with just two surfaces, at least for this power!

The performance of the plano-convex +4.00 design, shown in figure 2.4, is illustrated in Figure 2.5b. When the eye views along the optical axis of the lens the power of the lens is indeed +4.00D. When the eye rotates through 30° from the optical axis, however, the real effect of the lens is +4.25D in the sagittal meridian and +5.25 in the tangential meridian. We can express this effect as being equivalent to a power +4.25D sphere with a +1.00D cylinder. This is so different from the paraxial power that it cannot be ignored! Clearly the choice of a plano-convex design for a lens of power +4.00D is a poor one.

In general, the surface powers which are chosen for any given lens are those which make the power obtained during oblique gaze as close as possible to the power obtained when the eye looks along the optical axis of the lens. We shall see that although we cannot make a +4.00D lens for which the power remains the same for all directions of gaze, we can certainly improve upon the performance given in Figure 2.5b.

Distortion

The aberration distortion affects the shape of the image rather than its sharpness and is also caused by the fact that the power of a spherical surface increases towards the periphery. Instead of remaining constant, the magnification increases more and more as the eye uses wider and wider zones of a spherical lens. Airy² showed at the beginning of the nineteenth century that distortion could be eliminated, only by correcting spherical aberration between the entrance and exit pupils of a system, thereby satisfying his celebrated Tangent Condition.

Figure 2.6 shows the effects of distortion on a square grid target such as graph paper (Figure 2.6a) viewed through plus and minus lenses. Plus lenses produce *pincushion* distortion (Figure 2.6b) and this type of distortion is typically seen when a strong plus lens used as a magnifier. Note that in addition to obtaining the characteristic pincushion-shaped image, the impression is also obtained that the object being viewed is concave, the centre of the object being further from the eye than the edges. Minus lenses produce *barrel* distortion (figure 2.6c) and is often reported by myopes who view through peripheral zones of their lenses. Note the convex appearance of the target afflicted with barrel distortion, the centre of the target appears to be closer than the edges.

That distortion is very sensitive to lens form will be seen immediately by the reader with a strong plano-convex lens. If the undistorted square grid object of Figure 2.6a is viewed with the lens held close to the eye, and first, with the plane surface nearer the eye, the amount of distortion is much less than when the lens is held with the curved surface closer to the eye. It can also be confirmed that when the plano-convex lens is used as a hand-held magnifier, close to the object being viewed but now at some distance from the eye, the distortion is much less when the curved surface faces the eye than when the plane surface faces the eye. It will be understood that the lens is now reversed from its usual position in front of the eye. When the form of a lens is changed, the amount of distortion that is exhibited by the lens also changes and may be the chief cause of the perceptual problems which occur when a subject is given new lenses of different form. The sources of these effects is discussed later.

Best form spectacle lenses

A *best form* spectacle lens is one whose surface powers have been specially computed to eliminate, or at least minimise, certain stated defects in its image forming properties.

It has already been pointed out that transverse chromatism is a function of the *V*-value of the lens material and will be minimised for a given power by selecting a material with the highest available *V*-value.

Any further improvement on chromatism can only be made by constructing an achromatic doublet, that is, a pair of lenses bonded together, in which the chromatism of one component is designed to neutralise the chromatism of the second. Such devices are too bulky to be seriously considered as spectacle lenses!

For most people, the brain readily adapts to distortion and usually, this aberration is an ongoing problem only in cases where there has been a significant change in lens form or a significant change in prescription.

The aberrations that remain over which the designer can exert some influence are oblique astigmatism and curvature of field.

The control which the designer can exercise over these two aberrations is illustrated in Figure 2.7, which shows how the off-axis performance of +4.00D lenses varies for three meniscus forms with base curves -6.00D, -4.50D and -4.00D.

In Figure 2.7a, the lens has been bent into a form where the oblique astigmatism has been entirely eliminated. Such a form is described as a *point-focal* lens form, from the German word *Punktal*, which means point-forming, and is, of course, the name still used by Carl Zeiss to describe their classic series of point-focal lenses.

At 35°, the power of the lens has dropped to +3.75D, that is, when the astigmatism is fully corrected, the mean oblique power of the lens changes by -0.25D. We say that the lens has a *mean oblique error* at 35° of -0.25D.

If the form of the lens is flattened from the point-focal bending, the tangential power decreases and for the -4.50 bending depicted in Figure 2.7b is now the same as the back vertex power of the lens. Such a form is described as a *minimum tangential error* form and is seen to suffer from an ever-increasing amount of aberrational astigmatism, albeit small, as the eye rotates away from the optical axis. The oblique astigmatic error amounts to about +0.25D at 35° and the blurring effect of this small cylinder is certain to be less than the 0.25 sphere blur found in the point-focal form depicted in Figure 2.7a.

In Figure 2.7c the bending of the lens has been reduced still further to a -4.00D base curve and it can be seen in the field diagram, that the tangential and sagittal oblique vertex sphere powers have increased to just the point where the focal lines within the eye would lie either side of, and equidistant from, the retina. At 35° the off-axis power of the lens is +3.85DS/+0.30DC, the tangential power is +0.15D too great and the sagittal power 0.15D too weak compared with the paraxial power. The mean oblique power of the lens is +4.00D. This form of lens is known as a *Percival lens* design and is free from mean oblique error for the zone in question.

Figure 2.8 illustrates field diagrams for -4.00D lenses made in point-focal form (+5.00 base curve), minimum tangential error form (+3.87 base curve) and Percival form (+3.25 base curve).

Tscherning's Ellipses

Modern spectacle lens design is undertaken with the aid of a computer which can perform high-speed, exact trigonometric ray tracing through a lens and produce field diagrams such as those depicted in Figures 2.7 and 2.8, in a matter of seconds, rather than days, as was the case when ray traces were performed by hand. In an attempt to speed up lens design in the days before the computer, it was customary to derive approximate equations for best form lenses using, so-called, third-order theory where the trigonometric functions of the angles were replaced by a power series which gave reasonable results for angles up to about 15°. Such methods were applied to spectacle lens design during the 19th century notably by Airy (1827)² and Tscherning (1904)³. Tscherning, in particular, pointed out that the relationship between the form and power of a spectacle lens, according to third-order calculations, was quadratic in nature. Whitwell⁴ later showed that if the solutions to Tscherning's equations were plotted graphically they would form an ellipse which is known now as *Tscherning's Ellipse*. Such ellipses can be constructed for point-focal, Percival or minimum tangential error forms, and for both distance or near vision designs.

Figure 2.9 illustrates Tscherning's Ellipse constructed for point-focal forms, used for distance vision. The lenses are made from CR 39 material of refractive index 1.498 and mounted 27 mm in front of the eye's centres of rotation.

The ellipse shows very nicely that lenses can be made free from astigmatism only over a certain range of powers (+7.25 to -22.25 in the case of the ellipse illustrated in Figure 2.9) and that within this range there are two forms for each power. The shallower form, and the form usually supplied in practice, is known as the Ostwalt point-focal form and the steeper form as the Wollaston point-focal form.

Lens powers outside the range depicted in Figure 2.9 cannot be made free from oblique astigmatism when restricted to the use of spherical surfaces. It will be seen later, that powers outside the range of the Ellipse can be made in point-focal form by the use of an aspherical surface which can be designed to neutralize the astigmatism of oblique incidence.

When Tscherning's Ellipses are constructed for higher refractive index media, it is found that there is hardly any change in the upper limit for the plus range of powers but that the lower limit for minus lenses does increase, as shown in Figure 2.10, which compares ellipses constructed for materials of refractive index, 1.50, 1.70 and 1.90.

It can also be seen from Figure 2.10 that, as a general rule, when the refractive index increases, the bending of the lens also needs to increase, in order to eliminate astigmatism.

When the third-order results given by Tscherning's Ellipses are compared with those obtained by accurate trigonometric ray tracing it is found that the third-order results are somewhat too steep. The tendencies, however, which are so graphically depicted by the Ellipses, are the same as those for lens forms which have been determined by accurate means. There are generally two forms of lens within a certain range of powers which can be corrected for oblique astigmatism.

Dispensing problems due to lens form

As a general rule, it is wise to dispense lenses in best-form curvatures for then, not only are off-axis errors minimised but different series do not differ much in form from one another and the distortion is about the same for a given power.

It is well-known that myopes, in particular, who become accustomed to a particular lens form are often dissatisfied when their lens form is changed and usually the only solution to their problem is to provide a new pair of lenses in a similar form to their previous pair.

Typically, a myope may have become accustomed to wearing flat-form lenses and although, in theory, curved-form lenses provide better off-axis performance, many do not tolerate a change from their flat-form design. An idea of the difference in performance between flat-form and curved-form lenses can be obtained by comparing the differences in off-axis performance of the following pairs of lenses (Table 2.1).

Table 2.1
Comparison of optical performance of -4.00D and -10.00D lenses made in different forms

-4.00 lens				-4.00 lens			
+5.00 base form				plano-concave form			
	OAE	MOE	Dist	OAE	MOE	Dist	
10°	0	+0.01	0.3%	-0.08	-0.06	0.4%	
20°	+0.01	+0.02	1.4%	-0.32	-0.24	2.3%	
30°	-0.01	+0.15	3.7%	-0.77	-0.56	6.2%	
40°	-0.10	+0.36	7.8%	-1.46	-1.01	4.4%	
-10.00 lens				-10.00 lens			
+2.00 base form				plano-concave form			
	OAE	MOE	Dist	OAE	MOE	Dist	
10°	-0.01	+0.02	0.9%	-0.07	-0.04	1.1%	
20°	-0.03	+0.09	4.3%	-0.28	-0.15	5.0%	
30°	+0.01	+0.28	11.3%	-0.61	-0.32	13.5%	
40°	+0.23	+0.70	25.2%	-1.03	-0.51	32.2%	

Considering first the -4.00D lens forms, it can be seen that in the case of the +5.00D base form, as the eye rotates away from the optical axis of the lens, the off-axis power decreases as the mean oblique error becomes positive. The eye is undercorrected in oblique gaze and accommodation will only worsen the situation.

For the plano-concave form, on the other hand, the off-axis power increases, the mean oblique error being negative and the eye is over-corrected in oblique gaze. If the eye accommodates, however, it can sharpen the image, especially if the eye is viewing an object whose principal features are at right angles to the axis of the oblique astigmatism.

Also, the distortion, which is of the barrel variety, increases at a faster rate for the flat-form lens, it is virtually double that of the best-form design.

Similar conclusions can be drawn from the data given for the -10.00D lens forms but it will be realised that numerically, the changes are less dramatic than for the weaker power lens. Of course, this is because the change in form is less dramatic for the -10.00D lens. Nevertheless, the off-axis power becomes positive for the curved-form lens whereas it becomes negative for the plano-concave form as the eye rotates away from the optical axis of the lenses.

The chief cause of the perceptual problems that these differences in off-axis performance produce, is not well-understood. An obvious explanation is the large difference in off-axis powers of the two lens forms illustrated in the table above. However, the symptoms usually described by subjects imply a

change in retinal image shape. Certainly they suffer a large change in distortion as evidenced by the values given in the Table 2.1, and it could be that it is really the change in distortion which is the underlying source of complaint. Certainly, in the absence of any history of problems associated with a change in lens form, when a subject is happily wearing an unexpected lens form, such as a -4,00D lens in plano-concave form, it is sensible, in the absence of any symptoms, not to change the form from that to which the subject has become accustomed.

Changes in form for higher index materials

Inspection of Figure 2.10 will confirm that when the refractive index of the lens material changes, the lens form does need to change in order to provide the same off-axis powers. Table 2.2 indicates the form required for various lens powers which have approximately the same off-axis performance as plano-concave forms made in spectacle crown glass or CR 39 material. Needless to say, the transverse chromatism exhibited by the lens is dependent upon the V-value of the material and will only match if the V-value of the new material matches that of crown glass or CR 39.

**Table 2.2
Form of higher index lenses**

Lens Power	original front curve (1.498/1.523)	required front curves when new refractive index is			
		1.600	1.700	1.800	1.900
-4.00	plano	+0.25	+0.50	+0.75	+1.00
-6.00	plano	+0.25	+0.75	+1.25	+1.62
-8.00	plano	+0.37	+0.87	+1.50	+2.00
-10.00	plano	+0.37	+1.00	+1.75	+2.25
-12.00	plano	+0.37	+1.00	+1.75	+2.50

Toric lens forms

Astigmatic lenses are normally supplied in curved form for the same reason as spherical lenses, to improve the off-axis performance of the lens. A *toroidal* surface can be imagined to be a cylindrical surface whose axis meridian is also curved, and a *toric* lens is simply a curved lens with one toroidal surface (figure 2.11). Three different types of toroidal surface are used in ophthalmic lens manufacture, the *tyre-formation* surface (Figure 2.11a), which lends itself well to mass-production techniques, the *barrel-formation* surface which is better suited to individual surface working, and the rarer *capstan-formation* surface (Figure 2.11c).

The lower numerical curve on the toroidal surface is called the *base curve* and the steeper curve, the *cross curve*.

For example the prescription +4.00/+2.00 x 90 could be supplied in either of the forms given below:
 front curve +8.00 x 180/ +10.00 x 90, back curve -4.00
 or, front curve +9.00, back curve -3.00 x 90/-5.00 x 180.

The first of these two forms is described as a +8.00 base toric lens and the second, as a -3.00 base toric lens.

Stock glass single vision uncut lenses are usually made in plus base toric form, employing a tyre-formation surface, since they are easier to mass-produce in this form. Stock plastics single vision uncuts are usually made in minus base toric form. Singly-worked toric lenses could be surfaced in plus base form but today, in general, the preferred method of working by the surfacing workshop is to use a minus base toric form. This method of working has several advantages from the workshop's point of view. Single vision lenses can be processed using the same tools as those required for bifocal lenses where the segment or progressive surface is on the front of the blank. Also the cylinder power of the tool is the same as the cylinder power which is required for the lens. The cross curve does not need any compensation for the thickness of the lens as it would in the case of a plus-base toric design. Finally, if the blank manufacturer provides a reasonable compensation for lens thickness appropriate to the curve under consideration the minus-base toric form permits the fewest number of surfacing tools to be maintained by the workshop.

From an optical point of view, plus-base torics give slightly better performance for plus prescriptions and low-power minus prescriptions whereas, in moderate and high myopia the optical performance is better when the cylinder is incorporated on the back surface, the lenses being supplied as a minus-

base toric designs. Ideally, the toroidal surface should be of barrel-form which is possible with singly worked lenses if toroidal tools are maintained by the surfacing house in barrel-form. Minus-base toric construction also improves the appearance of the lens since the variation in edge thickness that occurs along different power meridians, can be hidden on the back of the lens. For very deep minus lenses it is easier for the manufacturer to incorporate the cylinder on the front surface of the lens in the form of a plano-convex cylindrical surface.